Chapter 4.

Magnetic Circuit Analysis

Topics to cover:

Introduction

For performance prediction of electromagnetic devices, magnetic field analysis is required. Analytical solution of field distribution by the Maxwell's equations, however, is very difficult or sometimes impossible owing to the complex structures of practical devices. Powerful numerical methods, such as the finite difference and finite element methods, are out of the scope of this subject. In this chapter, we introduce a simple method of magnetic circuit analysis based on an analogy to dc electrical circuits.

A Simple Magnetic Circuit

Consider a simple structure consisting of a current carrying coil of N turns and a magnetic core of mean length l_c and a cross sectional area A_c as shown in the diagram below. The permeability of the core material is m_c. Assume that the size of the device and the operation frequency are such that the displacement current in Maxwell's equations are negligible, and that the permeability of the core material is very high so that all magnetic flux will be confined within the core. By Ampere's law,
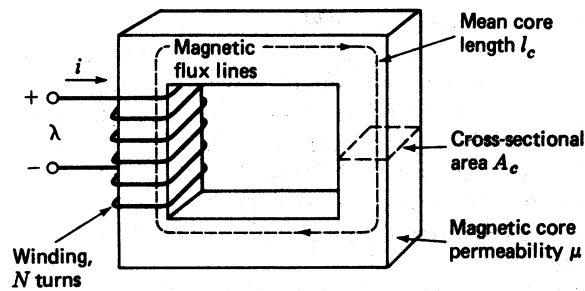
$$\oint_C \mathbf{H} \bullet d\mathbf{l} = \oint_S \mathbf{J} \bullet d\mathbf{a}$$

we can write $H_c l_c = Ni$

where H_c is the magnetic field strength in the core, and Ni the ***magnetomotive force***. The magnetic flux through the cross section of the core can expressed as

$$f_c = B_c A_c$$

A simple magnetic circuit

where f_c is the flux in the core and B_c the flux density in the core. The constitutive equation of the core material is
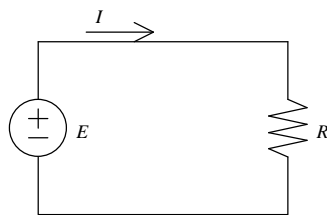
$$B_c = m H_c$$

Therefore, we obtain
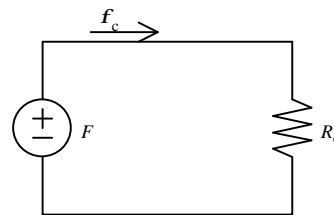
$$f_c = \frac{Ni}{l_c / (m_c A_c)} = \frac{F}{R_c}$$

If we take the magnetic flux f_c as the "current", the magnetomotive force $F=Ni$ as the "emf of a voltage source", and $R_c = l_c/(m_c A_c)$ (known as the **magnetic reluctance**) as the "resistance" in the magnetic circuit, we have an analog of **Ohm's law** in electrical circuit theory.

Electric Circuit	**Magnetic Circuit**

$$I = \frac{E}{R}$$

$$f_c = \frac{F}{R_c}$$

Magnetic Circuital Laws

Consider the magnetic circuit in the last section with an air gap of length l_g cut in the middle of a leg as shown in figure (a) in the diagram below. As they cross the air gap, the

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magnetic flux lines bulge outward somewhat as illustrate in figure (b). The effect of the *fringing* field is to increase the effective cross sectional area A_g of the air gap. By Ampere's law, we can write

$$F = Ni = H_c l_c + H_g l_g$$

where

$$H_c l_c = \frac{B_c}{m_c} l_c = \frac{f_c}{m_c A_c} l_c = f_c R_c$$
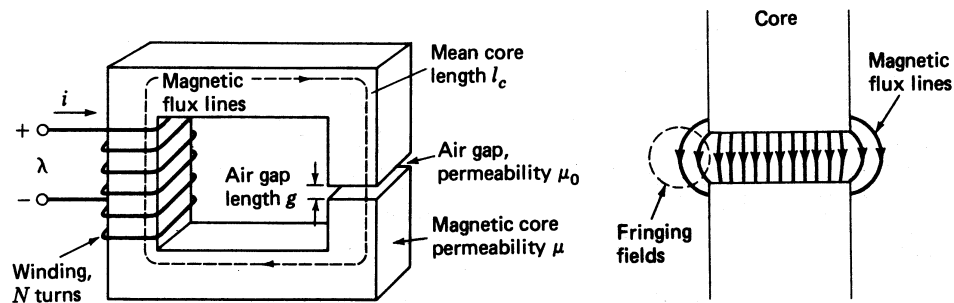
A simple magnetic circuit with an air gap

and

$$H_g l_g = \frac{B_g}{m_b} l_g = \frac{f_g}{m_b A_g} l_g = f_g R_g$$
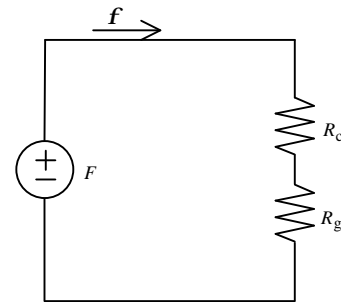
According to Gauss' law in magnetics,

$$\oint_S \mathbf{B} \bullet d\mathbf{a} = 0$$

we know

$$f_c = f_g = f$$

Series magnetic circuit

Therefore,

$$F = \left(R_c + R_g \right) f$$

That is, the above magnetic circuit with an air gap is analogous to a series electric circuit. Further, if we regard $H_c l_c$ and $H_g l_g$ as the "voltage drops" across the reluctance of the core and airgap respectively, the above equation from Ampere's law can be interpreted as an analog to the *Kirchhoff's voltage law* (KVL) in electric circuit theory, or
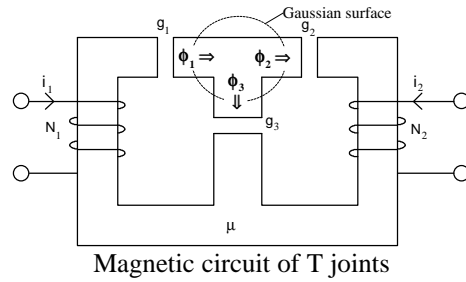
$$\sum R_k f_k = \sum F_k$$

The *Kirchhoff's current law* (KCL) can be derived from the Gauss' law in magnetics. Consider a magnetic circuit as shown below. When the Gauss' law is applied to the T joint in the circuit, we have

$$\sum_{k=1}^{3} \boldsymbol{f}_k = 0$$

or in general,

$$\sum_{k=1}^{n} \boldsymbol{f}_k = 0$$

Magnetic circuit of T joints

Having derived the Ohm's law, KVL and KCL in magnetic circuits, we can solve very complex magnetic circuits by applying these basic laws. All electrical dc circuit analysis techniques, such as mesh analysis and nodal analysis, can also be applied in magnetic circuit analysis.

For nonlinear magnetic circuits where the nonlinear magnetization curves need to be considered, the magnetic reluctance is a function of magnetic flux since the permeability is a function of the magnetic field strength or flux density. Numerical or graphical methods are required to solve nonlinear problems.
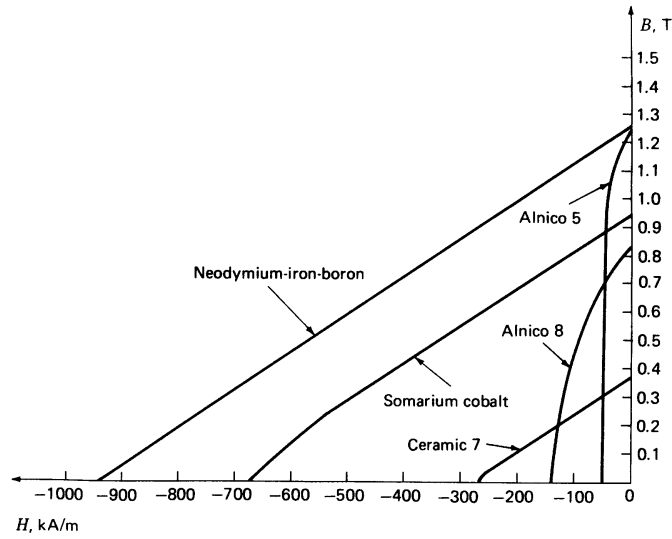
Magnetic Circuit Model of Permanent Magnets

Permanent magnets are commonly used to generate magnetic fields for electromechanical energy conversion in a number of electromagnetic devices, such as actuators, permanent magnet generators and motors. As mentioned earlier, the characteristics of permanent magnets are described by demagnetization curves (the part of hysteresis loop in the second quadrant). The diagram below depicts the demagnetization curve of five permanent magnets. It can be seen that the demagnetization curves of some most commonly used permanent magnets: Neodymium Iron Boron (NdFeB), Samarium Cobalt, and Ceramic 7 are linear. For the convenience of analysis, we consider the magnets with linear demagnetization curves first.

Consider a piece of permanent magnet of a uniform cross sectional area of A_m and a length l_m. The demagnetization curve of the magnet is a straight line with a coercive force of H_c and a remanent flux density of B_r as shown below. The demagnetization curve can be expressed analytically as

$$B_m = \frac{B_r}{H_c}\left(H_m + H_c\right) = \boldsymbol{m}_m\left(H_m + H_c\right)$$

where $\boldsymbol{m}_m = B_r/H_c$ is the permeability of the permanent magnet, which is very close to $\boldsymbol{m}_0$, the permeability of free space. For a NdFeB magnet, $\boldsymbol{m}_m = 1.05\boldsymbol{m}_0$.

Demagnetization curves of permanent magnets
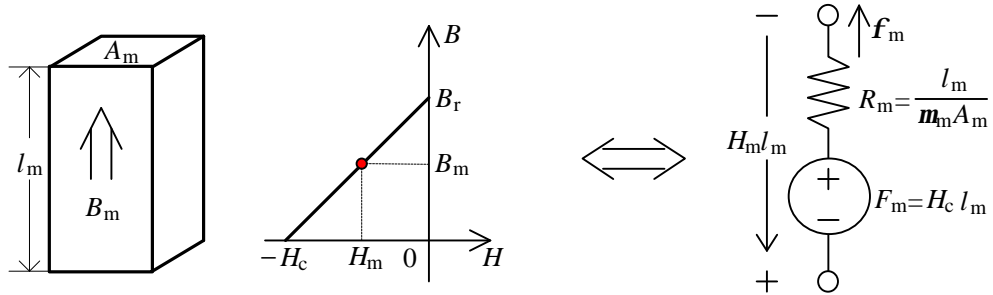
Magnetic circuit model of a magnet with linear demagnetization curve

The magnetic "voltage drop" across the magnet can be expressed as

$$H_m l_m = \left(\frac{B_m}{\mu_m} - H_c \right) l_m = \frac{l_m}{\mu_m A_m} f_m - H_c l_m = R_m f_m - F_m$$

where $R_m = \dfrac{l_m}{\mu_m A_m}$ is the reluctance and $F_m = H_c l_m$ the magnetomotive force ("voltage source") of the magnet. It should be noted that in the magnet, $\mathbf{B}_m$ and $\mathbf{H}_m$ are in opposite directs.

For a magnet with a nonlinear demagnetization curve, the above magnetic circuit model is still valid, except that the magnetic permeability becomes

$$\mu_m = \frac{B_m}{H_m + H_c}$$

which is a function of the magnetic field in the magnet. Notice that H_m is a negative value since it is in the opposite direction of $\mathbf{B}_m$. The derivation for the magnetic circuit model of a nonlinear magnet is illustrated graphically by the diagram below.
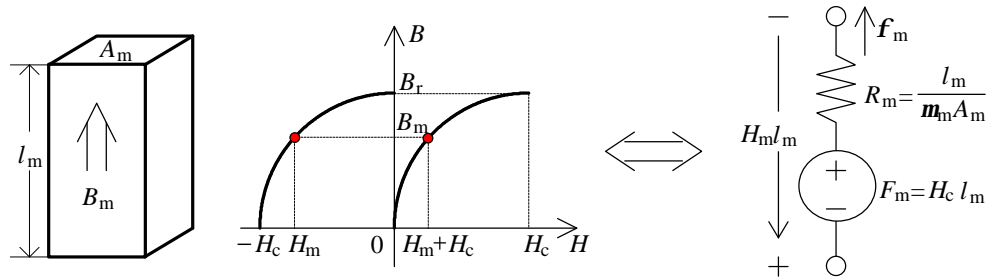
Magnetic circuit model of a magnet with nonlinear demagnetization curve

It should also be understood that the operating point (H_m, B_m) will not move along the nonlinear demagnetization curve if a small (such that the magnet will not be demagnetized) periodic external magnetic field is applied to the magnet. Instead, the operating point will move along a minor loop or simply a straight line (center line of the minor loop) as illustrated in the diagram on the right hand side.
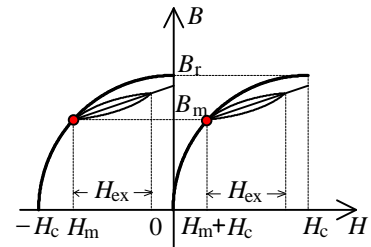
Movement of operating point of a nonlinear magnet under an external field H_{ex}

Inductance

Consider a two coil magnetic system as shown below. The magnetic flux linkage of the two coils can be express as

$$\lambda_1 = \lambda_{11} + \lambda_{12} \qquad \text{and} \qquad \lambda_2 = \lambda_{21} + \lambda_{22}$$

where the first subscript indicates the coil of flux linkage and the second the coil carrying current. By defining the self and mutual inductances of the two coils as

$$L_{jk} = \frac{\lambda_{jk}}{i_k} \qquad (j=1,2 \text{ and } k=1,2)$$

where L_{jk} is the self inductance of the jth coil when $j=k$, the mutual inductance between the jth coil and the kth coil when $j \neq k$, and $L_{jk} = L_{kj}$, the flux linkages can be expressed as

$$\lambda_1 = L_{11}i_1 + L_{12}i_2 \qquad \text{and} \qquad \lambda_2 = L_{21}i_1 + L_{22}i_2$$

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The above definition is also valid for a *n* coil system. For a linear magnetic system, the above calculation can be performed by switching on one coil while all other coils are switched off such that the magnetic circuit analysis can be simplified. This is especially significant for a complex magnetic circuit. For a nonlinear magnetic system, however, the inductances can only be calculated by the above definition with all coils switched on.
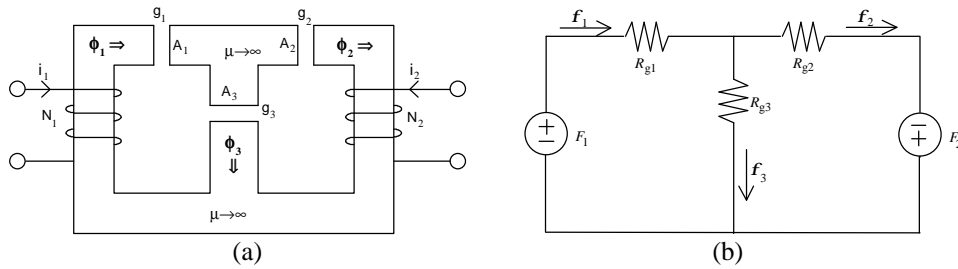
(a)   (b)

Magnetic circuit of a two coil system

Electromotive Force

When a conductor of length *l* moves in a magnetic field of flux density **B** at a speed **v**, the induced electromotive force (*emf*) can be calculated by

$$\mathbf{e} = l\mathbf{v} \times \mathbf{B}$$

For a coil linking a time varying magnetic field, the induced *emf* can be calculated from the flux linkage of the coil by

$$e_k = \frac{d\boldsymbol{l}_k}{dt} = \sum_{j=1}^{n} \frac{d\boldsymbol{l}_{kj}}{dt} = \sum_{j=1}^{n} L_{kj} \frac{di_j}{dt} \qquad\qquad (k=1,2,...n)$$

Magnetic Energy

In terms of inductance, the magnetic energy stored in an *n* coil system can be expressed as

$$W_f = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \boldsymbol{l}_{jk} i_j = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \frac{\boldsymbol{l}_{jk}\boldsymbol{l}_{kj}}{L_{jk}} = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} L_{jk} i_j i_k$$

Exercises

1. A coils of 200 turns is wound uniformly over a wooden ring having a mean circumference of 600 mm and a uniform cross sectional area of 500 mm^2. If the current through the coil is 4 A, calculate:
   (a) the magnetic field strength,
   (b) the flux density, and
   (c) the total flux

   Answer: 1333 A/m, 1675×10^{-6} T, 0.8375 mWb

2. A mild steel ring having a cross sectional area of 500 m^2 and a mean circumference of 400 mm has a coil of 200 turns wound uniformly around it. Calculate:
   (a) the reluctance of the ring and
   (b) the current required to produce a flux of 800 μWb in the ring. (Given that m_r is about 380).

   Answer: 1.677×10^6 A/Wb, 6.7 A

3. Fig.Q3 shows an iron circuit with a small air gap cut in it. A 6000 turn coil carries a current I=20 mA which sets up a flux within the iron and across the air gap. If the iron cross section is 0.8×10^{-4} m^2, the mean length of flux path in iron is 0.15 m, μ_r=800 in iron and air gap length is 0.75 mm, calculate the air gap flux density. It may be assumed that the flux lines flow straight across the air gap, i.e. air gap cross section is also 0.8×10^{-4} mm^2.

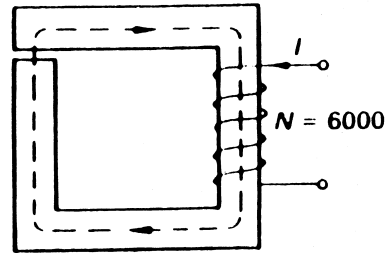   Fig.Q3

   Answer: 0.16 T

4. A magnetic circuit is made of mild steel arranged as in Fig.Q4. The center limb is wound with 500 turns and has a cross sectional area of 800 mm^2. Each of the outer limbs has a cross sectional area of 500 mm^2. The air gap has a length of 1 mm. Calculate the current required to set up a flux of 1.3 mWb in the center limb, assuming no magnetic leakage and fringing. The mean lengths of the various magnetic paths are shown on the diagram. (Use the given B-H curve).
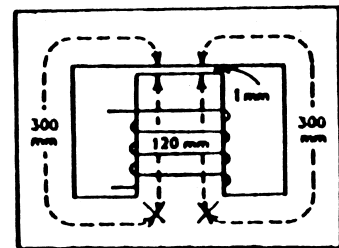

   Fig.Q4

   Answer: 4 A

5. A magnetic circuit is made up of steel laminations shaped as in Fig.Q5. The width of the iron is 40 mm and the core is built up to a depth of 50 mm, of which 8 percent is taken up by insulation between the laminations. The gap is 2 mm long and the effective area of the gap is 2500 mm^2. The coil is wound with 800 turns. If the leakage factor (the ratio of the total flux linking the coil over the air gap flux) is 1.2, calculate the magnetizing current required to produce a flux of 0.0025 Wb across the air gap. (Use the given B-H curve).
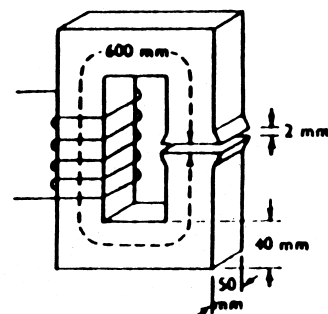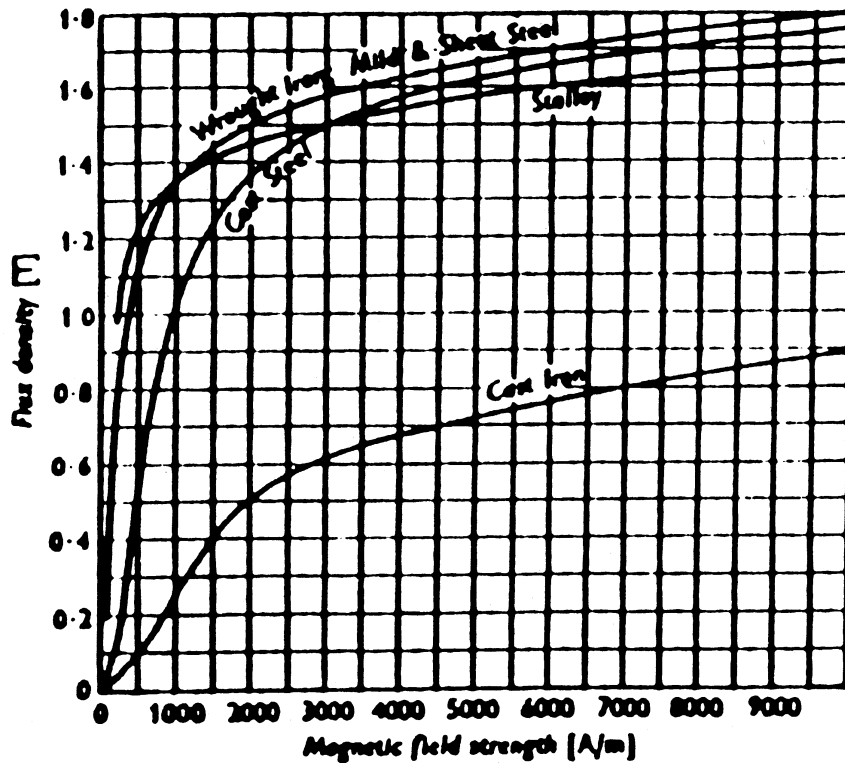

   Fig.Q5

   Answer: 5 A

6. It is desired to achieve a time varying magnetic flux density in the air gap of the magnetic circuit of Fig.Q6(a) of the form

$$B_g = B_0 + B_1 \sin \omega t$$

where $B_0 = 0.5$ T and $B_1 = 0.25$ T. The dc field B_0 is to be created by a NdFeB permanent magnet, whereas the time varying field is to be created by a time varying current. Assume the permeability of the iron is infinite and neglect the fringing effect.

(a) For the air gap dimensions given in Fig.Q6(a), find the magnet length d if the magnet area A_m equals the air gap area A_g. Fig.Q6(b) gives the demagnetization curve of NdFeB permanent magnet.

(b) Find the excitation current required to achieve the desired time varying air gap flux density.

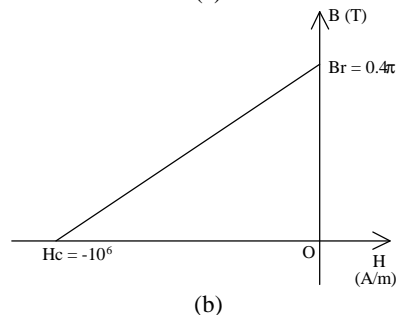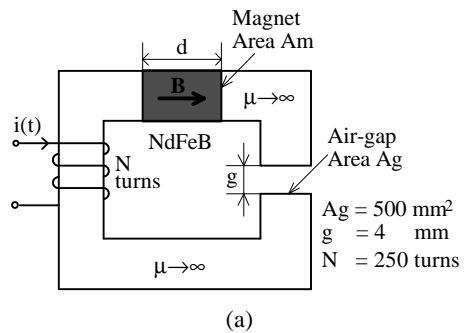*Answer: 2.64 (mm), i = 5.28sinωt (A)*

(a)

(b)

Fig.Q6 (a) Magnetic circuit of Problem 1, (b) Demagnetization curve of permanent magnet NdFeB

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7. Fig.Q7 shows a magnetic circuit with air gaps $g_1 = g_2 = g_3 = 1$ mm and coils $N_1 = 100$ turns and $N_2 = 200$ turns. The cross sectional area A of the circuit is 200 mm^2. Assume the permeability of the core material approaches infinity and the fringing effect is negligible.
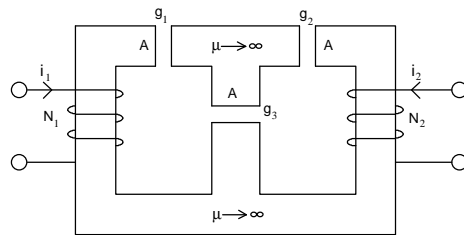

Fig.Q7 Magnetic circuit of Problem 7

Calculate:

(a) the self and mutual inductances;

(b) the total magnetic energy stored in the system, if the currents in the coils are $i_1 = i_2 = 1$ A;

(c) the mutual inductance between N_1 and N_2, if the air gap g_3 is closed.

Answer: 1.676 mH, 6.702 mH, 1.676 mH, 5.865 $\times 10^{-3}$ J, 0